TRINITY COLLEGE

YEAR 11
MATHEMATICS
SPECIALIST

Test 1, 2023
Section One: Calculator Free
Counting Techniques & Vectors I

STUDENT'S NAME: MARKING KEY [KRISZYK]

DATE: Thursday 16th March   **TIME:** 25 minutes   **MARKS:** 27
ASSESSMENT %: 10

INSTRUCTIONS:
Standard Items: Pens, pencils, drawing templates, eraser
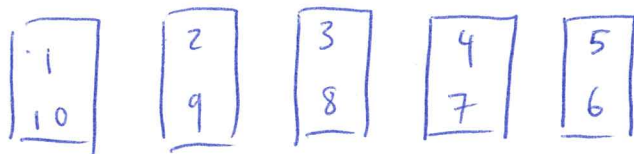Special Items:

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

Question 1

(2 marks)

Explain that if six distinct numbers from the integers 1 to 10 are chosen, then there will be two of them that sum to 11.

pigeonholes → integers pairs that sum to 11
pigeons → distinct integers

1	2	3	4	5
10	9	8	7	6

First 5 digits will mean every pigeonhole is half selected

1 more digit must be selected making a pair which sums to 11.

Question 2 **(7 marks)**

If $a = 3i - 2j$ and $b = -3i + j$ and $c = xi - j$, determine the following:

(a) $2a + 3b$ (1 mark)

$$\begin{pmatrix} 6 \\ -4 \end{pmatrix} + \begin{pmatrix} -9 \\ 3 \end{pmatrix} = \begin{pmatrix} -3 \\ -1 \end{pmatrix} \quad \text{or} \quad -3\underset{\sim}{i} - \underset{\sim}{j}$$

(b) a vector in the same direction as b but the same magnitude as a. (3 marks)

$$|\underset{\sim}{b}| = \sqrt{10} \quad \checkmark$$

$$|\underset{\sim}{a}| = \sqrt{13} \quad \checkmark$$

$$\underset{\sim}{r} = \frac{\sqrt{13}}{\sqrt{10}} \begin{pmatrix} -3 \\ 1 \end{pmatrix}$$

$$= \frac{\sqrt{130}}{10} \begin{pmatrix} -3 \\ 1 \end{pmatrix} \quad \checkmark$$

(c) the value of x if $|c| = |a|$. (3 marks)

$$|\underset{\sim}{a}| = \sqrt{13}$$

Allow F/T

$$|\underset{\sim}{c}| = \sqrt{x^2 + 1^2} \quad \checkmark$$

$$x^2 + 1 = 13$$

$$x^2 = 12$$

$$x = \pm 2\sqrt{3} \quad \checkmark$$

$$\checkmark$$
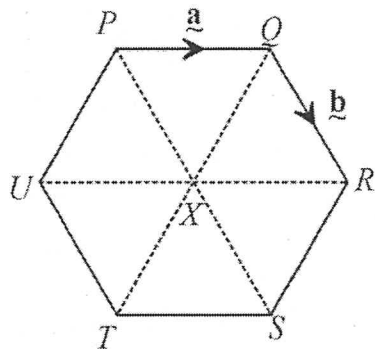
Question 3   (4 marks)

PQRSTU is a regular hexagon where $\overrightarrow{PQ} = \underset{\sim}{a}$ and $\overrightarrow{QR} = \underset{\sim}{b}$. Express the following vectors in terms of $\underset{\sim}{a}$ and/or $\underset{\sim}{b}$.

(a) $\overrightarrow{QX}$ (2 marks)

$$= \overrightarrow{QP} + \overrightarrow{PX}$$

$$= -\underset{\sim}{a} + \underset{\sim}{b} \quad \checkmark\checkmark$$

(b) $\overrightarrow{SQ} \quad = \overrightarrow{SX} + \overrightarrow{XQ} \quad \checkmark$ (2 marks)

$$= -\underset{\sim}{b} + -(\overrightarrow{QX})$$

Allow F/T

$$= -\underset{\sim}{b} - (-\underset{\sim}{a} + \underset{\sim}{b})$$

$$= \underset{\sim}{a} - 2\underset{\sim}{b} \quad \checkmark$$

Question 4   (3 marks)

The point P divides the line segment from M (-3,3) to N (13, -9) in the ratio 1:3. Determine the position vector of point P.
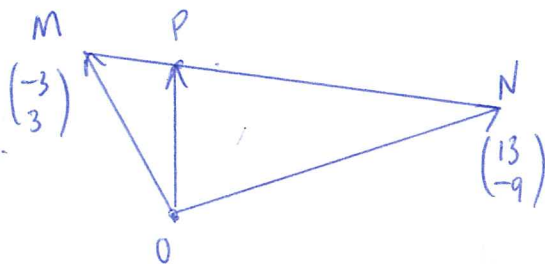
$$\overrightarrow{MN} = \overrightarrow{MO} + \overrightarrow{ON}$$

$$= -\begin{pmatrix} -3 \\ 3 \end{pmatrix} + \begin{pmatrix} 13 \\ -9 \end{pmatrix}$$

$$= \begin{pmatrix} 16 \\ -12 \end{pmatrix} \quad \checkmark$$

$$\overrightarrow{MP} = \frac{1}{4} \overrightarrow{MN}$$

$$= \begin{pmatrix} 4 \\ -3 \end{pmatrix} \quad \checkmark$$

$$\overrightarrow{OP} = \overrightarrow{OM} + \overrightarrow{MP}$$

$$\overrightarrow{OP} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \checkmark \qquad = \begin{pmatrix} -3 \\ 3 \end{pmatrix} + \begin{pmatrix} 4 \\ -3 \end{pmatrix}$$

Question 5 (6 marks)

(a) Given the vectors $\mathbf{a} = -2\mathbf{i} + 3\mathbf{j}$, $\mathbf{b} = \mathbf{i} - 4\mathbf{j}$ and $\mathbf{c} = 5\mathbf{i} + 2\mathbf{j}$, determine:

(i) $\mathbf{c} \cdot 2\mathbf{a}$ (2 marks)

$$= 2(\mathbf{a} \cdot \mathbf{c}) \qquad\qquad = 2(-10 + 6)$$
$$= 2\begin{pmatrix} -2 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 5 \\ 2 \end{pmatrix} \checkmark \qquad = -8 \quad \checkmark$$

(ii) k if $\mathbf{a} \cdot (3\mathbf{i} - k\mathbf{j}) = 9$ (2 marks)

$$\begin{pmatrix} -2 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -k \end{pmatrix} = 9$$

$$-6 - 3k = 9 \quad \checkmark$$

$$-3k = 15$$

$$k = -5 \quad \checkmark$$

(b) Vectors $\mathbf{p} = 4\mathbf{i} - 3\mathbf{j}$ and $\mathbf{q} = a\mathbf{i} + 3\mathbf{j}$ are perpendicular to each other. Determine the possible value(s) of a. (2 marks)

$$\begin{pmatrix} 4 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} a \\ 3 \end{pmatrix} = 0 \quad \checkmark$$

$$4a - 9 = 0$$

$$a = \frac{9}{4} \quad \checkmark$$

Question 6   (5 marks)

Given $\mathbf{g} = 4\mathbf{i} + \mathbf{j}$ and $\mathbf{k} = 2\mathbf{i} - 5\mathbf{j}$, express $\mathbf{a} = -10\mathbf{i} - 8\mathbf{j}$ in terms of $\mathbf{g}$ and $\mathbf{k}$.

$$\lambda \mathbf{g} + \mu \mathbf{k} = -10\mathbf{i} - 8\mathbf{j}$$

$$\lambda \begin{pmatrix} 4 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -5 \end{pmatrix} = \begin{pmatrix} -10 \\ -8 \end{pmatrix} \quad \checkmark$$

$$4\lambda + 2\mu = -10$$
$$\lambda - 5\mu = -8 \qquad \checkmark$$

$$\lambda = -8 + 5\mu$$

$$4(-8 + 5\mu) + 2\mu = -10$$
$$22\mu = 22$$
$$\mu = 1 \qquad \checkmark$$

$$\therefore \lambda = -3 \qquad \checkmark$$

$$\therefore \mathbf{a} = -3\mathbf{g} + \mathbf{k} \qquad \checkmark$$

END OF QUESTIONS

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Test 1, 2023
Section Two: Calculator Allowed
Counting Techniques & Vectors I

STUDENT'S NAME: MARKING KEY [KRISZYK]

DATE: Thursday 16th March   **TIME:** 25 minutes   **MARKS:** 29
ASSESSMENT %: 10

INSTRUCTIONS:
Standard Items: Pens, pencils, drawing templates, eraser
Special Items: 1 A4 page notes, Classpad, Scientific Calculator

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

Question 7 (7 marks)

Rohan has 6 different Mathematics books and 5 different English books. He picks 4 of his favourite Mathematics books and 3 of his favourite English books to put on a new bookshelf.

(a) How many ways can the books be arranged on the bookshelf? (2 mark)

$$\binom{6}{4} \times \binom{5}{3} \times 7! = 756\,000 \quad \checkmark\checkmark$$

(b) How many ways can the books be arranged if the English books must be arranged in alphabetical order on the shelf? (Assume each has a different title) (2 marks)

$$\binom{6}{4} \times \binom{5}{3} \times 5! = 18000 \quad \checkmark\checkmark$$

(c) Determine how many ways the seven books be arranged on the bookshelf if a Mathematics book must be in the furthest left position. (3 marks)

$$6 \times \binom{5}{3} \times \binom{5}{3} \times 6! = 432\,000 \quad \checkmark\checkmark\checkmark$$
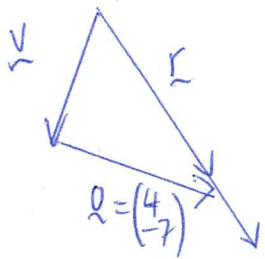
Question 8   **(6 marks)**

A navy vessel can travel at a steady speed of 80 km/h in still water. The navy vessel needs to travel directly from port C to port D, where $\overline{CD}$ = (60**i** - 140**j**) km.

There is an ocean current of velocity (4**i** – 7**j**) km/h.

(a)   Determine the velocity vector, in the form (a**i** + b**j**), which the captain should set so that the vessel travels directly from port C to port D.   **(5 marks)**

$$\underset{\sim}{v} = \begin{pmatrix} a \\ b \end{pmatrix} \qquad |\underset{\sim}{v}| = 80$$

$$\therefore \sqrt{a^2 + b^2} = 80$$

$$\underset{\sim}{r} = \begin{pmatrix} a \\ b \end{pmatrix} + \begin{pmatrix} 4 \\ -7 \end{pmatrix}$$

$$\underset{\sim}{\ell} = \begin{pmatrix} 4 \\ -7 \end{pmatrix}$$

$$\therefore \begin{pmatrix} 60 \\ -140 \end{pmatrix} = t \begin{pmatrix} a+4 \\ b-7 \end{pmatrix}$$

$$\text{via CP} \quad \underset{\sim}{r} = \begin{pmatrix} 30.67 \\ -73.89 \end{pmatrix}$$

✓ diagram
✓ eqn mag of v
✓ eqn for r
✓ solves on CP
✓ velocity vector

(b)   Determine the time (to nearest minute) it takes to make the journey from port C to port D.   **(1 mark)**

$$t = 1.73 \text{ hrs}$$

$$t = 1 \text{ hr } 44 \text{ mins.}$$

Question 9   **(7 marks)**

A subcommittee of six people is to be chosen from the following fourteen people.

Abe, Ben, Chris, Dave, Eric, Frances, George, Hal, Ida, Jake, Kym, Lal, Mike and Nic.

How many different subcommittees are possible in each of the following cases:

(a)   There are no restrictions as to the makeup of the subcommittee.   (1 mark)

$$\binom{14}{6} = 3003 \quad \checkmark$$

(b)   Abe and Ben must both be on the subcommittee.   (1 mark)

$$\binom{2}{2} \times \binom{12}{4} = 495 \quad \checkmark$$

(c)   Jake and Kym must either both be on the subcommittee or neither be on the subcommittee.
                                                                                                        (2 marks)

Both + Neither

495 + 924 $\checkmark$

= 1419 $\checkmark$

(d)   Frank and Ida must not both be on the subcommittee.   (2 marks)

$$\binom{14}{6} - 495 = 2508 \quad \checkmark\checkmark$$

Question 10   **(5 marks)**

The position vectors of points P, Q and R are $3i - 2j$, $-2i - j$ and $-5i + 3j$ respectively. Determine the magnitude of $\overrightarrow{QP}$ and $\overrightarrow{QR}$ hence the size of angle PQR.

$$\overrightarrow{QP} = \overrightarrow{QO} + \overrightarrow{OP}$$

$$= -\begin{pmatrix} -2 \\ -1 \end{pmatrix} + \begin{pmatrix} 3 \\ -2 \end{pmatrix}$$

$$= \begin{pmatrix} 5 \\ -1 \end{pmatrix} \checkmark$$

$$\overrightarrow{QR} = \overrightarrow{QO} + \overrightarrow{OR}$$

$$= -\begin{pmatrix} -2 \\ -1 \end{pmatrix} + \begin{pmatrix} -5 \\ 3 \end{pmatrix}$$

$$= \begin{pmatrix} -3 \\ 4 \end{pmatrix} \checkmark$$

$$|QP| = \sqrt{26}$$

$$|QR| = 5$$

both $\checkmark$

$$\cos\theta = \frac{\begin{pmatrix} 5 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} -3 \\ 4 \end{pmatrix}}{5\sqrt{26}} \checkmark$$

$$= \frac{-15 - 4}{5\sqrt{26}}$$

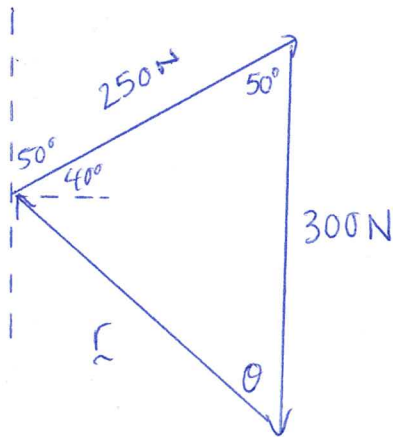$$\theta = 138.18° \checkmark$$

Question 11   **(4 marks)**

Three forces are applied to a body. One has a magnitude of 300 N and acts due South. Another has a magnitude of 250 N and acts on a bearing of 050°.

If all three forces are in equilibrium determine the magnitude and direction of the third force.

$$|\underset{\sim}{r}| = \sqrt{250^2 + 300^2 - 2(250)(300)\cos 50°}$$

Magnitude = 236.8 N

(diagram with 250 N, 50°, 40°, 300 N, θ, r labelled)

$$\frac{\sin\theta}{250} = \frac{\sin 50}{236.8}$$

$$\theta = 53.97°$$
$$\sim 54°$$

Bearing = 360 - 54
        = 306° T

✓ diagram

✓ Magnitude

✓ Value for θ

✓ Bearing.

Accept 143.97° from Horizontal

END OF QUESTIONS